# Teacher Note

# Learning and Assessing Multiplication Combinations

Students began working on the multiplication combinations in Grade 3, learning combinations with products to 50. They continued on to the rest of the combinations (to $12 \times 12$) in the first Grade 4 unit, *Factors, Arrays, and Multiples.* In this unit, students practice and review multiplication combinations to $12 \times 12$ as homework, making use of these combinations as they solve multiplication problems in class. The goal is for all students to become fluent with multiplication combinations by the end of this unit. To meet this goal, however, some students will need additional practice during the unit, and some will need continuing practice after the unit is completed. This Teacher Note provides recommendations for supporting students in this ongoing practice.

## Why Do We Call Them *Combinations*?

The pairs of factors from $1 \times 1$ through $12 \times 12$ are traditionally referred to as "multiplication facts"—those multiplication combinations with which students are expected to be fluent. The *Investigations* curriculum follows the National Council of Teachers of Mathematics (NCTM) convention of calling these expressions *combinations* rather than *facts. Investigations* does this for two reasons. First, naming *only* particular addition and multiplication combinations as *facts* seems to give them elevated status, more important than other critical parts of mathematics. In addition, the word *fact* implies that something cannot be learned through reasoning. For example, it is a fact that the first president of the United States was George Washington, and it is a fact that Rosa Parks was born in Alabama in 1913. If these facts are important for us to know, we can remember them or use reference materials to look them up. However, the product for the multiplication combination $6 \times 7$ can be determined in many ways; it is logically connected to our system of numbers and operations. If we forget the product, but understand what multiplication is and know some related multiplication combinations, we can find the product through reasoning. For example, if we know that $5 \times 7 = 35$, we can add one more 7 to determine that the product of $6 \times 7$ is 42. If we know that $3 \times 7 = 21$, we can reason that the product of $6 \times 7$ would be twice that, $2 \times (3 \times 7) = 42$.

The term *facts* does convey a meaning that is generally understood by some students and family members, so you will need to decide whether to use the term *facts* along with *combinations* in certain settings in order to make your meaning clear.

## Fluency with Multiplication Combinations

Like NCTM, this curriculum supports the importance of students learning the basic combinations through a focus on reasoning about number relationships: "Fluency with whole-number computation depends, in large part, on fluency with basic number combinations—the single digit addition and multiplication pairs and their counterparts for subtraction and division. Fluency with basic number combinations develops from well-understood meanings for the four operations and from a focus on thinking strategies. . . ." (*Principles and Standards for School Mathematics*, pages 152–153)

Fluency means that combinations are quickly accessible mentally, either because they are immediately known or because the calculation that is used is so effortless as to be essentially automatic (in the way that some adults quickly derive one combination from another).

## Helping Students Learn the Multiplication Combinations

### A. Students Who Know Their Combinations to 50

Students who know their combinations to 50, as well as the combinations that involve multiplying by 10 up to 100 ($6 \times 10$, $7 \times 10$, $8 \times 10$, $9 \times 10$, $10 \times 10$), can work on learning the most difficult combinations. Here is one way of sequencing this work.

***1. Learning the remaining combinations with products to 100.*** There are 6 difficult facts to learn (other than the ×11 and ×12 combinations, which are, in fact, not as difficult as these, and are discussed below). These six difficult combinations are: $6 \times 9$ (and $9 \times 6$), $7 \times 8$ (and $8 \times 7$), $7 \times 9$ (and $9 \times 7$), $8 \times 8$, $8 \times 9$ (and $9 \times 8$), and $9 \times 9$. Note that knowing that multiplication is commutative is crucial for learning all the multiplication combinations. The work with Array Cards supports this understanding, see **Teacher Note:** Representing Multiplication with Arrays, page 117.

Students can work on one or two of these most difficult multiplication combinations each week. Make sure that they use combinations they do know to help them learn ones they don't know—for example, $8 \times 7 = 2 \times (4 \times 7)$, or $9 \times 7 = (10 \times 7) - 7$. They can write these related multiplication combinations as "start with" hints on the Multiplication Cards. If most of your class needs to work on the same few hard combinations, you might want to assign the whole class to focus on two of these each week.

***2. Learning the ×11 and ×12 combinations.*** We consider these combinations to be in a different category. Historically, these combinations were included in the list of "multiplication facts." However, when we are dealing with 2-digit numbers in multiplication, an efficient way to solve them is through applying the distributive property, breaking the numbers apart by place as you would with any other 2-digit numbers. We include them here because some local or state frameworks still require knowing multiplication combinations through $12 \times 12$. In addition, 12 is a number that occurs often in our culture, and it is useful to know the ×12 combinations fluently. Most students learn the ×11 combinations easily because of the pattern (11, 22, 33, 44, 55, . . .) created by multiplying successive whole numbers by 11. They should also think through why this pattern occurs: $3 \times 11 = (3 \times 10) + (3 \times 1) = 30 + 3 = 33$. They should think through why $11 \times 10 = 110$ and $11 \times 11 = 121$ by breaking up the numbers. Students can learn the ×12 combinations by breaking the 12 into a 10 and 2, e.g., $12 \times 6 = (10 \times 6) + (2 \times 6)$. Some students may also want to use doubling or adding on to known combinations: $12 \times 6 = 2 \times (6 \times 6)$, or $12 \times 6 = (11 \times 6) + 6$.

### B. Students Who Need Review and Practice of Combinations to 50

Students who have difficulty learning the multiplication combinations often view this task as overwhelming—an endless mass of combinations with no order and reason. Bringing order and reason to students' learning of these combinations in a way that lets them have control over their progress is essential. Traditionally, students learned one "table" at a time (e.g., first the ×2 combinations, then the ×3 combinations, the ×4 combinations, and so on). However, the multiplication combinations can be grouped in other ways to support learning related combinations.

First, make sure that students know all multiplication combinations that involve ×0, ×1, ×2, ×5, and ×10 (up to $10 \times 10$) fluently. (Students worked with the ×0 combinations in Grade 3.) Note that, although most fourth graders can easily count by 2, 5, and 10, the student who is fluent does not need to skip count to determine the product of multiplication combinations involving these numbers.

When students know these combinations, turn to those that they have not yet learned. Provide a sequence of small groups of combinations that students can relate to what they already know. There are a number of ways to do this.

***1. Learning the ×4 combinations.*** Work on the ×4 combinations that students do not yet know: 3 × 4, 4 × 4, 6 × 4, 7 × 4, 8 × 4, and 9 × 4. Help students think of these as doubling the ×2 combinations. So, 4 × 6 = (2 × 6) + (2 × 6), or 4 × 6 = 2 × (2 × 6). Students may verbalize this idea as "4 times 6 is 2 times 6 and another 2 times 6," or "to get 4 times 6, I double 2 × 6." Doubling is also useful within the ×4 combinations; for example, when students know that 3 × 4 = 12, then that fact can be used to solve 6 × 4: 6 × 4 = (3 × 4) + (3 × 4). Getting used to thinking about doubling with smaller numbers will also prepare students for using this approach with some of the harder combinations.

***2. Learning the square numbers.*** Next, students learn or review the four remaining combinations that produce square numbers less than 50: 3 × 3, 5 × 5, 6 × 6, and 7 × 7. These are often easy for students to remember. If needed, use doubling or a known combination for "start with" clues during practice (e.g., 6 × 6 is double 3 × 6; 5 × 5 is 5 more than 4 × 5). Students can also build these combinations with tiles or draw them on grid paper to see how they can be represented by squares.

***3. Learning the remaining combinations with products to 50.*** Finally, learn or review the six remaining combinations with products to 50: 3 × 6 through 3 × 9, 7 × 6, and 8 × 6. First, relate them to known combinations (e.g., double 3 × 3 or halve 6 × 6 to get 3 × 6), and then practice them.

## Assessing Students' Knowledge of Multiplication Combinations

In Investigation 3 of this unit, students are assessed on their fluency with the multiplication combinations. For this assessment, students are expected to be able to solve 30 problems that are representative of the set of combinations to 12 × 12, with accuracy, in three minutes. If they can solve them all within that time limit, students are either accessing these combinations from memory or they are able to make a very quick calculation that is almost automatic. Some students may take longer than others to reach this level of fluency. You can expect to have students in your class who may need to do this assessment more than once. They will continue to identify combinations they still need to work on and will practice those with their Multiplication Cards. These students may also use Array Cards for practice, either by laying them out factor side up and identifying the products or by playing *Missing Factors*. You may have other favorite practice methods or activities that you want to suggest for particular students. Also, enlist parents or other family members to help with this practice.

In this unit, students also begin to work on division problems that are counterparts to the multiplication combinations. They will continue to work on these in the next multiplication and division unit, *How Many Packages? How Many Groups?* In the meantime, as students work on solving division problems, help them relate division expressions to the multiplication combinations they know; for example, what multiplication combination can help you solve 24 ÷ 6?

## Fluency Benchmarks for Learning Facts Through the Grades

**Addition:** Fluent by end of Grade 2, with review and practice in Grade 3

**Subtraction:** Fluent by end of Grade 3, with review and practice in Grade 4

**Multiplication:** Fluent with multiplication combinations with products to 50 by the end of Grade 3; up to 12 × 12 by the middle of Grade 4, with continued review and practice

**Division:** Fluent by end of Grade 5